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Calculation of PID Controller Parameters for Unstable First Order Time Delay Systems

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Abstract— In this paper, a numerical approach for the fractional order proportional-integral-derivative controller (FO-PID) design for the unstable first order time delay system is proposed. The controller design is based on the system time delay. In order to obtain the relation between the controller parameters and the time delay, for several amounts of the plant time delay and the fractional derivative and integral orders, the ranges of stabilizing controller parameters are determined. First, for a typical time delay plant and the fractional order controller, the D-decomposition technique is used to plot the stability region(s). The controller derivative gain has been considered as one. By changing the fractional derivative and integral orders, a small amount in each stage, some ranges of proportional and integral gains are achieved which stabilize the system, independent of the fractional λ , μ orders. Therefore a set of different controllers for any specified time delay system is obtained. This trend for several various systems with different values of time delay has been done and the proportional and integral gains of the stabilizing controller have been calculated. Then we have fitted these values to the exponential functions and the proportional and integral gains have been obtained in terms of the system time delay. Using these relations, we can specify some ranges of the proportional and integral gains and obtain a set of stabilizing controllers for any given system with certain time delay. In these relations, fractional derivative and integral orders haven't part, and therefore can be applied to any fractional order controller design (for $0.1 \le \lambda, \mu \le 0.9$). Thus we have reached a numerical approach from the graphical D-decomposition method. In this method, there is freedom in choosing the values of λ and μ (they can fall in the range of [0.1, 0.9]), and there is no need to plot the stability boundaries and check the different regions to determine the stable one. This numerical method does not offer the complete set of the stabilizing controllers. Whenever the system time delay is more, the specified range of proportional and integral gains will be smaller. In other words, the extent of obtained stability region is inversely proportional to the system time delay. Finally, the introduced numerical approach is used for stabilizing an unstable first order time delay system.

Index Terms—Fractional order PID controller, numerical approach, time delay.

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1 Introduction

ALTHOUGH great advances have been achieved in the control science, the proportional-integral-derivative controller is still the most used industrial controller.

According to the Japan Electric Measuring Instrument Manufacturers' Association in 1989, PID controller is used in more than 90% of control loops [1], [2]. As an example for the the application of PID controllers in industry, slow industrial processes can be pointed, low percentage overshoot and small settling time can be obtained by using this controller [1]. Widespread application of the PID controller is due to the simple and implementable structure and its robust performance in the wide range of the working conditions [3], [4]. This controller provides feedback, it has the ability to eliminate steady-state offsets through integral action, and it can anticipate the future through derivative action. The mentioned benefits have caused widespread use of the PID controllers. The derivative action in the control loop will improve the damping, and therefore by accelerating the transient response, a higher proportional gain can be obtained. Precise attention must be paid to setting the derivative gain because it can amplify high-frequency noise. In this paper, for the fractional order PID controller design, the derivative gain (k_d) is set 1, that will result in design simplicity. Most available commercial PID controllers have a limitation on the derivative gain [2]. During the past half century, many theoretical and industrial studies have been done in PID controller setting rules and stabilizing methods [3]. So far several different techniques have been proposed to obtain PID controller parameters and the research still continues to improve the system performance and increase the control quality. Ziegler and Nichols in 1942 proposed a method to set the PID controller parameters. Hagglund and Astrom in 1995, and Cheng- Ching in 1999, introduced other techniques [5]. By generalizing the derivative and integral orders, from the integer field to non-integer numbers, the fractional order PID controller is obtained. In fractional order PID controller design, there is more freedom in selecting the parameters and more flexibility in their setting . This is due to posse of choice -both integer and non-integer numbers- for integral and derivative orders. Therefore control requirements will be easier to comply [6], [7].

Before using the fractional order controllers in design, an introduction to fractional calculus is required. Over 300 years have passed since the fractional calculus has been introduced. The first time, calculus generalization to fractional, was proposed by Leibniz and Hopital for the first time and afterwards, the systematic studies in this field by many researchers such as Liouville (1832), Holmgren (1864) and Riemann (1953) were performed [8].

Fractional calculus is used in many fields such as electrical transmission losses systems and the analysis of the mechatronic systems. Some controller design techniques are based on the classic PID control theory generalization [7]. Due to the recent advances in the fractional calculus field and the emergence of fractance electrical element, the fractional order controller implementation has become more feasible [6], [9], [10]. Consequently, fractional order PID controller analysis and synthesis have received more attention [11], [12], [13], [14], [15], [16]. Results obtained from various articles published in this field, indicate that the fractional order PID controllers enhance the stability, performance and robustness of the feedback control system [6], [11], [12]. Maiti, Biswas and Konar [1] have significantly reduced the overshoot percentage, the rise and settling times, compared to classic PID controller, using the fractional order PID controller. Applying the fractional order PID controller ($PI^{\lambda}D^{\mu}$), the system dynamic characteristics can be adjusted better [17]. Many dynamic processes can be described by a first order time delay transfer function [18]. The need to control time delay processes can be found in different industries such as rolling mills. Varying time delay process control becomes difficult using classical control methods [19]. Simple formulas are available for setting the PID controller parameters for the stable first order time delay system, but when the system is unstable, the problem will be more difficult and therefore the unstable systems control requires more attention. Many attempts have been made in field of their stabilization [20], [21], [22], [23], [24]. So far, various design techniques have been suggested for the fractional order controller design [13], [14], [25], [26]. It has been shown that fractional order PID controllers have a better performance comparing to integer order ones, for both integer and fractional order control systems.

In the controller design for an unstable system, the most important design issue is stabilizing the closed-loop system [6]. As an example of previous research in stabilizing the unstable processes, we can point to De Paor and O'Malley research in 1989, which discussed unstable open loop system stabilization with a PID or PD controller [23]. Hamaci [3] has concluded that fractional order PID controller has a better response than classic one. In this paper, a numerical method is introduced to design the fractional order controllers for any unstable first order system with specified time delay.

2 The Fractional Order PID Controller Design

2.1 A Review to Design Methods

Hamamci and Koksal [4] have designed the fractional order PD controller to stabilize the integration time delay system, which result that stability region extent is reversed with the system time delay. Maiti, Biswas, and Konar, in 2008, could significantly reduce the overshoot percentage, the rise time, and the settling time by using fractional order PID controllers. They introduced PSO (particle swarm optimization) optimization technique for the fractional order PID controller design. In their method, the controller has been designed based on required maximum overshoot and the rise time. In the mentioned technique, the closed loop system characteristic equation is minimized in order to get an optimal set of the controller parameters [1]

One of the methods to obtain the complete set of stabilizing PID controllers is plotting the global stability regions in the (k_p, k_i, k_d) -space, which is called the D-decomposition technique [3], [4], [6], [8]. This technique is used in both fractional and integer order systems analysis and design.

Cheng and Hwang [6] has designed the fractional order proportional - derivative controller to stabilize the unstable first order time delay system and D- decomposition method has been used. The graphical D- decomposition technique results for such systems are simple.

The D- decomposition technique can be used for fractional order time delay systems and fractional order chaos systems. In this method, the stability region boundaries are obtained, which are described by real root boundary (RRB), infinite root boundary (IRB), and complex root boundary (CRB). By crossing these boundaries in the (k_p, k_i, k_d)-space, several regions will be achieved. By choosing an arbitrary point from each region and checking its stability, the region's stability is tested. If the selected point is stable, the region including that point would be stable, and if the selected point is not stable then the region would be unstable. By obtaining the stability boundaries and plotting the stability regions, a complete set of stabilizing fractional order controller parameters is obtained. The mentioned algorithm is simple and effective.

2.2 The D-decomposition Technique

In general, the characteristic equation of the fractional order closed loop system is defined as

$$P(s) = p_k s^{q_k} + p_{k-1} s^{q_{k-1}} + \ldots + p_1 s^{q_1} + p_0. \tag{1}$$

In P parameter space, the boundaries between stable and unstable regions are defined by three following parts:

Real root boundary (RRB): A real root crosses over the imaginary axis at s=0. Thus the real root boundary is obtained by setting s = 0 in (1). RRB is determined as $p_0 = 0$.

Complex root boundary (CRB): A pair of complex roots, cross over the imaginary axis at $s = j\omega$

Infinite root boundary (IRB): An infinite real root crosses over the imaginary axis at $s = j\infty$ Therefore IRB line is obtained by putting $p_k = 0$ in (1). First, by using the D-decomposition graphical method, stability boundaries and then stability region(s), are obtained. RRB and IRB lines are given by

$$RRB : k_i = 0 \tag{2}$$

$$IRB : k_d = \begin{Bmatrix} 0 & \mu > 1 \\ \infty & \mu \le 1 \end{Bmatrix}. \tag{3}$$

Then by writing k_p and k_i equations in terms of k_d , λ , and μ , the CRB curve equation is derived. The transfer function of the plant and fractional order PID controller and closed-loop system characteristic equations have been given in (4), (5), and (6), respectively. Fig. 1 shows the block diagram of the closed loop system.

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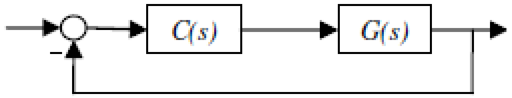

Fig. 1. The closed loop system block diagram.

$$G(s)=\frac{e^{-\theta s}}{\tau s-1},\tau=1 \tag{4}$$

$$C(s)=k_p+\frac{k_i}{s^{\lambda}}+k_d s^{\mu} \tag{5}$$

$$P(s)=(\tau s-1)s^{\lambda}+e^{-\theta s}\left(k_p s^{\lambda}+k_i+k_d s^{\lambda+\mu}\right)=0 \tag{6}$$

CRB curve equations are given in (7).

$$CRB: P(\omega;k)=0$$
$$CRB:\left\{\begin{array}{l} k_p=\dfrac{A(\omega)B(\omega)-C(\omega)D(\omega)+k_d\left(E(\omega)B(\omega)-F(\omega)D(\omega)\right)}{G(\omega)B(\omega)-D(\omega)H(\omega)} \\ k_i=\dfrac{C(\omega)G(\omega)-A(\omega)H(\omega)+k_d\left(F(\omega)G(\omega)-E(\omega)H(\omega)\right)}{G(\omega)B(\omega)-D(\omega)H(\omega)} \end{array}\right\} \tag{7}$$

Where

$$A(\omega)=\left(\cos\frac{\lambda\pi}{2}\right)\omega^{\lambda}+\tau\left(\sin\frac{\lambda\pi}{2}\right)\omega^{(\lambda+1)} \tag{8a}$$

$$B(\omega)=(-\sin\omega\theta) \tag{8b}$$

$$C(\omega)=\left(\sin\frac{\lambda\pi}{2}\right)\omega^{\lambda}-\tau\left(\cos\frac{\lambda\pi}{2}\right)\omega^{(\lambda+1)} \tag{8c}$$

$$D(\omega)=(\cos\omega\theta) \tag{8d}$$

$$E(\omega)=-\omega^{(\lambda+\mu)}\left(\cos(\omega\theta).\cos\frac{(\lambda+\mu)\pi}{2}+\sin(\omega\theta).\sin\frac{(\lambda+\mu)\pi}{2}\right) \tag{8e}$$

$$F=\omega^{(\lambda+\mu)}\left(-\cos(\omega\theta).\sin\frac{(\lambda+\mu)\pi}{2}+\sin(\omega\theta).\cos\frac{(\lambda+\mu)\pi}{2}\right) \tag{8f}$$

$$G(\omega)=(\cos\omega\theta).\left(\cos\frac{\lambda\pi}{2}\right)\omega^{\lambda}+(\sin\omega\theta).\left(\sin\frac{\lambda\pi}{2}\right)\omega^{\lambda} \tag{8g}$$

$$H(\omega)=(\cos\omega\theta).\left(\sin\frac{\lambda\pi}{2}\right)\omega^{\lambda}-(\sin\omega\theta).\left(\cos\frac{\lambda\pi}{2}\right)\omega^{\lambda} \tag{8h}$$

By plotting IRB and RRB lines and CRB curve in the (k_p,k_i)-plane, for fixed k_d, λ, and μ, as ω runs from 0 to ∞, Stability regions are obtained [3], [4], [6], [8]. Assuming $k_d=1$, the D-decomposition technique is used for various values of λ, μ and θ, and stability regions are obtained by arbitrary selected test points. In this paper, Nyquist stability test has been used to check the stability of the selected point of each region. The range of changes for λ and μ are considered between zero and 1.

3 Obtaining the Controller Parameters Based on the Plant Time Delay

To determine the relations between the controller parameters with time delay system, the stability regions are determined using D-decomposition technique. In Fig. 2 the stable region of $\frac{e^{-0.5s}}{s-1}$ plant with $PI^{0.4}D^{0.7}$ controller is marked in gray. Notice that the small region that includes small proportional and integral gains (near the origin) is not a part of the stability regions.

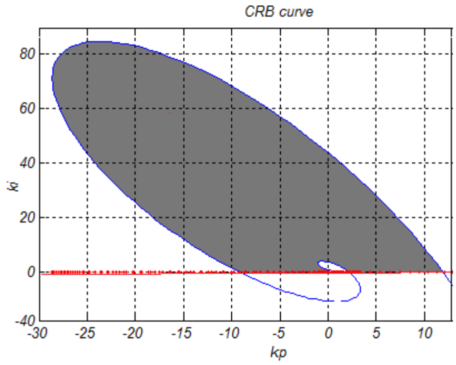

Fig. 2. The stability region for the $\frac{e^{-0.5s}}{s-1}$ plant and the $PI^{0.4}D^{0.7}$ controlle.

Two limited rectangles are selected from the stability region, one rectangle in the first quarter of the (k_p,k_i)-plane and another one in the second quarter of the plane. Fig. 3 shows two selected rectangles. Rectangle (1) which includes positive values of the proportional and integral gains is marked in light gray, and rectangular (2) which contains negative proportional gain and positive integral gain, is marked in dark gray. Two rectangles are chosen so that the minimum value of k_i is zero (means that rectangles have been placed on the RRB line) and the minimum value of k_p is close to zero as much as possible. Maximum value of the proportional gain in the second rectangle is selected near zero as much as possible.

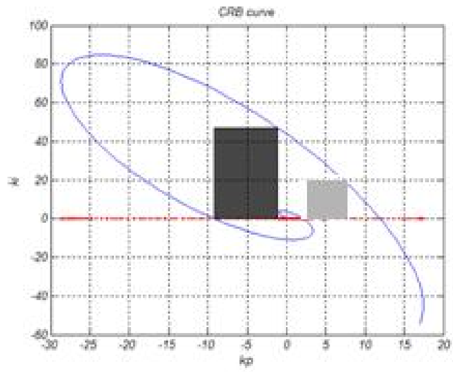

Fig. 3. Two selected rectangles, for the $\frac{e^{-0.5s}}{s-1}$ system, which is controlled by the $PI^{0.4}D^{0.7}$ controller.

Considering the mentioned criteria for selecting two stability rectangles, for various amounts of the system time delay, we have obtained the minimum and maximum values of k_p and maximum value of k_i. First, by considering different $PI^{\lambda}D^{\mu}$ controllers $(0<\lambda,\mu<1)$ for $\frac{e^{-0.1s}}{s-1}$

system, we choose two mentioned stability rectangles. In both rectangles, the maximum and minimum value of the proportional gain and the maximum value of the integral gain are shown with $k_{p_{\max}}$, $k_{p_{\min}}$ and $k_{i_{\max}}$, respectively. By changing the fractional derivative and integral orders, the position of the rectangles on the (k_p, k_i)-plane will change, but in all, some values of k_p and k_i are the same.

This procedure is performed in two stages. First, rectangle 1 is considered. For $\theta = 0.1$, considering several controllers with different fractional orders, it can be observed that if the controller proportional gain is selected between 15 and 40 and the integral gain between zero and 25, for any fractional derivative and integral orders between 0.1 and 0.9, the controller will stabilize the system. By changing the value of the system time delay, this process is repeated and for each system, some ranges of k_p and k_i are determined, any arbitrary controller with these obtained parameters (where the fractional orders λ and μ are between 0.1 and 0.9) can stabilize the system. For some first order systems with the time delay θ, the obtained ranges of stabilizing controller parameters are given in Table 1. These k_p and k_i values belong to rectangle 1 within the stable region and have been obtained independently of the fractional derivative and integral orders. To study time delay effect on the range of controller parameters, in each step θ has been changed slightly (0.05).

To obtain the proportional and integral gains range independent of fractional derivative and integral orders, we consider different values of λ and μ, which change slightly in each step (0.05). By increasing the system time delay, the values of the minimum and maximum proportional gain and maximum selected value of the integral gain become smaller, Table 1 also confirms this reduction.

Table 1. The stabilizing parameters ranges in rectangle 1(for any arbitrary $PI^{\lambda}D^{\mu}$ controller $(0.1 \le \lambda, \mu \le 0.9)$)

θ	$k_{p_{\min}}$	$k_{p_{\max}}$	$k_{i_{\max}}$
0.1	15	40	25
0.3	5	14	5
0.7	2	5	1.5
1	1.3	3.3	0.8
1.3	1.1	2.6	0.7
1.8	1	1.4	0.25

To obtain these ranges based on the system time delay, several different systems with $0.1 \le \theta \le 1$ have been considered. Different values of the time delay are considered from 0.1 up to 1, and the obtained minimum and maximum values of k_p and k_i are fitted to the exponential functions. In Fig. 4 the obtained proportional gain ranges (which will result the stabilizing controller, independently of the fractional derivative and integral orders) and also their fitting to the exponential functions are shown. The upper curve is obtained from the maximum proportional gain values fitting, and the underlying curve is obtained from fitting the integral gain minimum values. These curves should not exceed $\left[k_{p_{\min}}, k_{p_{\max}}\right]$.

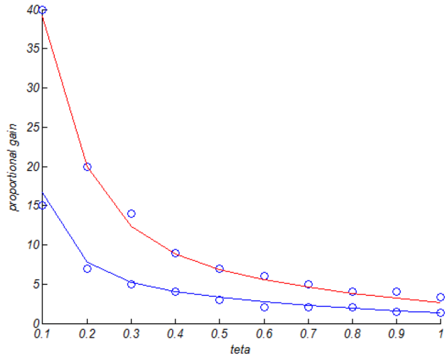

Fig. 4. The selected ranges for the proportional gain and the fitting results.

Maximum proportional gain in rectangle 1 is fitted to $a.e^{-b\theta} + c.e^{-d\theta}$, such that the resulting exponential function is close to the maximum value of k_p as much as possible (minimize the fitting error), also the fitting result should be located between the selected minimum and maximum proportional gain values. This fitting result is $76e^{-10.8\theta} + 16e^{-1.8\theta}$. Similarly, the minimum proportional gain in rectangle 1, is fitted to $45e^{-15\theta} + 8e^{-1.8\theta}$. The maximum integral gain which is selected from rectangle 1 is fitted to $60.1e^{-13\theta} + 8e^{-2.5\theta}$. According to these fitting results, for FO_PID controller design for the system with the time delay θ ($0.1 \le \theta \le 1$), selecting the proportional gain from (9), the integral gain from (10), and the fractional orders from the given range in (11), the closed-loop system would be stable.

$$k_p = a.e^{-b\theta} + c.e^{-d\theta}, \begin{cases} 45 \le a \le 76 \\ 11 \le b \le 15 \\ 8 \le c \le 16 \\ d = 1.8 \end{cases} \tag{9}$$

$$k_i = a.e^{-b\theta} + c.e^{-d\theta}, \begin{Bmatrix} 0 \le a \le 60 \\ 13 \le b \\ 0 \le c \le 8 \\ 3 \le d \end{Bmatrix} \tag{10}$$

$$0.1 \le \lambda, \mu \le 0.9 \tag{11}$$

Therefore, for any unstable first order system with time delay ($0.1 \le \theta \le 1$), the obtained exponential functions can be used to calculate the stabilizing controller parameters and to obtain a set of proportional and integral gains. These values can be used in any controller which its fractional derivative and integral orders are between 0.1 and 0.9, and therefore a wide set of the stabilizing controllers will be available. In this paper, the system time delay is considered smaller than or equal to the system time constant.

All obtained exponential functions, are only the functions of the system time delay, and are independent of the fractional controller derivative and integral orders. Now, we explain the system time delay effect on the position of rectangle2 in the (k_p, k_i)–plane and like before, we consider different values of the time delay and the derivative order and the integral order. For several values of the system time delay $(\theta \le 2)$, the position of rectangle 2 has been obtained and Table (2) shows the minimum and maximum proportional gain and maximum integral gain in rectangle 2.

Table2. The stabilizing parameters range in rectangle 2, for some delay system.

θ	$k_{p_{\min}}$	$k_{p_{\max}}$	$k_{i_{\max}}$
0.1	-45.7	-27	150
0.4	-10.5	-4.7	30
0.6	-6.7	-2.3	17
0.9	-4.3	-0.9	10
1.2	-3.1	-0.5	6.8
1.5	-2.4	-0.3	4.9
2	-1.9	0	3.1

Ranges which were obtained for the proportional and integral gains, are independent of λ and μ values, and for any λ and μ between [0.1, 0.9] the resultant controller will stabilize the closed loop system. When the system time delay increases, the minimum and maximum of k_p and maximum of k_i become smaller, as seen in Table 2. In controller design for the plant with specified delay, if the relations between stabilizing parameters and the plant time delay are given, using them a set of stabilizing controllers can be obtained. To get these relations, minimum and maximum of proportional gain and maximum of integral gain are fitted to the exponential functions of θ .The fitting results are given in Table 3.

Table3. The rectangle2 boundaries fitted to the exponential functions (independent of λ and μ)

$0.1 \le \theta \le 1; 0.1 \le \lambda, \mu \le 0.9$	
Fitting result of $k_{p_{\min}}$	$-\left(98e^{-12\theta} + 18.5e^{-1.7\theta}\right)$
Fitting result of $k_{p_{\max}}$	$-\left(48e^{-6.3\theta} + 2.1e^{-0.9\theta}\right)$
Fitting result of $k_{i_{\max}}$	$310e^{-12\theta} + 55e^{-2\theta}$

Using the exponential functions which are obtained from fitting the boundaries of the two selected rectangles, various ranges of stabilizing controllers can be obtained, and for the system with specified delay some values of stabilizing proportional and integral gains can be easily obtained.

For varying time delay systems, these exponential functions can be used to design the fractional order PID controller (with any arbitrary fractional derivative and integral order between 0.1 and 0.9). Since the obtained relations can be used in fractional controller design for $0.1 \le \lambda, \mu \le 0.9$, there are many alternatives in the controller choice. When the system time delay increases, the range of parameters would be smaller.

4 ILLUSTRATION

We consider an unstable first order plant which its transfer function is $\frac{e^{-0.34s}}{s-1}$. We use the introduced numerical method to obtain a set of stabilizing controller for this system. Some ranges of the proportional and integral gains are given in (12) .Here, both gains are positive.

$$k_{p_{\min}} = 45e^{-15\theta} + 8e^{-1.8\theta} = 4.62 \quad (12a)$$

$$k_{p_{\max}} = 76e^{-10.8\theta} + 16e^{-1.8\theta} = 10.6 \quad (12b)$$

$$k_{i_{\min}} = 0 \quad (12c)$$

$$k_{i_{\max}} = 60.1e^{-13\theta} + 8e^{-2.5\theta} = 4.1 \quad (12d)$$

Also, we can refer to Table 3 to calculate the controller parameters. According to this table, by choosing the proportional gain in the range [-12,-7.2], integral gain in the range [0, 33.1] and arbitrary λ and μ in the range [0.1, 0.9], a set of stabilizing controllers will be obtained.

5 CONCLUSION

In this paper, a numerical method has been proposed to design a fractional order PID controller for the unstable first order time delay system. In this method, some ranges of the proportional and integral gains are obtained based on the system time delay. If the proportional and the integral gains are selected from these ranges, the closed-loop system would be stable. The arbitrary fractional derivative and integral orders are selected from the range [0.1, 0.9]. The D-decomposition technique is used to derive the numerical relations. By introduced numerical method, without having to determine the stability boundaries and plotting them in the (k_p, k_i)-plane and checking the stability of all regions, a set of fractional order controller parameters for an unstable first order time delay system with transfer function $\frac{e^{-\theta s}}{s-1}$ is determined easily. In fractional controller design using the mentioned relations, derivative and integral orders can be chosen arbitrary numbers between 0.1 and 0.9. Although this approach does not get all the stabilizing controllers for the specified time delay system, but we have a simple design method because of simple calculations and the freedom to choose the fractional λ and μ orders. This method can also be used in varying time delay systems to obtain the proportional and integral gains as functions of the time delay, wherein using classical control methods will be difficult for these systems.

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